## The Principle of Mathematical Induction

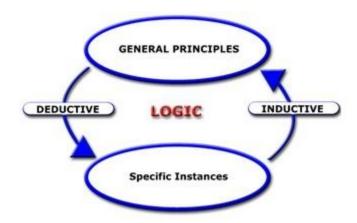
**Deduction**: Generalization of Specific Instance

Example: Rohit is a man & All men eat food, therefore, Rohit eats food.

**Induction:** Specific Instances to Generalization

Example: Rohit eats food. Vikash eats food. Rohit and Vikash are men. Then, All men eat food

Statement is true for n=1, n=k & n=k+1, then, the Statement is true for all natural numbers n.



## **Steps of Principle of Mathematical Induction:**

**Step 1:** Let P(n) be a result or statement formulated in terms of n(given question).

Step 2: Prove that P(1) is true

**Step 3:** Assume that P(k) is true

**Step 4:** Using Step 3, prove that P(k+1) is true

**Step 5:** Thus P(1) is true and P(k+1) is true whenever P(k) is true.

Hence, by the Principle of Mathematical Induction, P(n) is true for all natural numbers n.

**Example**: Prove that  $2^n > n$  for all positive integers n

**Solution:** 

Step 1: Let P(n):  $2^n > n$ 

Step 2: When  $n = 1, 2^1 > 1$ . Hence P(1) is true.

Step 3: Assume that P(k) is true for any positive integer k, i.e.,  $2^k > k$  ... (1)

Step 4: We shall now prove that P(k + 1) is true whenever P(k) is true.

Multiplying both sides of (1) by 2, we get

 $2* 2^k > 2*k$ 

i.e.,  $2^{k+1} > 2k$ 

or,  $2^{k+1} > k + k$ 

or,  $2^{k+1} > k+1$  (since k>1)

Therefore, P(k + 1) is true when P(k) is true. Hence, by principle of mathematical induction, P(n) is true for every positive integer n.